Further Pure 1 - June 2012

1 a) $\alpha + \beta = 7/5 \qquad \alpha\beta = 1/5$

b) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2}{\alpha\beta} + \dfrac{\beta^2}{\alpha\beta}$

$= \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$ *

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (7/5)^2 - 2(1/5) = 39/25$

$\therefore \dfrac{\alpha^2+\beta^2}{\alpha\beta} = \dfrac{39/25}{1/5} = 39/5$

c) $\boxed{\text{SUM}}$ $\alpha + 1/\alpha + \beta + 1/\beta = \alpha + \beta + 1/\alpha + 1/\beta$

$= (\alpha+\beta) + \dfrac{\alpha}{\alpha\beta} + \dfrac{\beta}{\alpha\beta}$

$= (\alpha+\beta) + \dfrac{\alpha+\beta}{\alpha\beta}$

$= 7/5 + \dfrac{7/5}{1/5} = 42/5$

$\boxed{\text{PRODUCT}}$ $(\alpha + 1/\alpha)(\beta + 1/\beta) = \alpha\beta + \alpha/\beta + \beta/\alpha + 1/\alpha\beta$

$= 1/5 + 39/5 + 1/1/5$

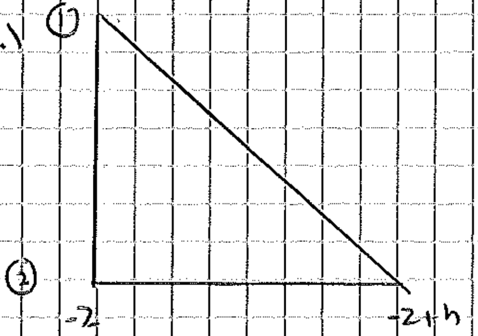
$= 13$

$x^2 - \boxed{\text{SUM}}\,x + \boxed{\text{PRODUCT}} = 0$

$\to x^2 - 42/5\,x + 13 = 0$

$\to 5x^2 - 42x + 65 = 0$

2 a)



$(1) = (-2)^4 + (-2) = 14$

$(2) = (-2+h)^4 + (-2+h)$

$= (-2)^4 + 4(-2)^3h + 6(-2)^2h^2$

$+ 4(-2)h^3 + h^3 - 2 + h$

$= 16 - 32h + 24h^2 - 8h^3 + h^4 - 2 + h$

$= h^4 - 8h^3 + 24h^2 - 31h + 14$

www.mymathscloud.com

$$\text{Gradient} = \frac{h^4 - 8h^3 + 24h^2 - 31h + 14 - 14}{h}$$

$$= h^3 - 8h^2 + 24h - 31$$

b) As $h \to 0$, grad $\to -31$

③ a) $z = x + iy$

$$\to i(x + iy + 7) + 3(x - iy - i)$$

$$= xi - y + 7i + 3x - 3iy - 3i$$

$$= 3x - y + xi - 3iy + 4i$$

$\boxed{\text{REAL}}$ $3x - y$

$\boxed{\text{IMAG}}$ $x - 3y + 4$

b) REAL = IMAG = 0

$\to$ ① $3x - y = 0$

② $x - 3y + 4 = 0$ or $x - 3y = -4$

② $\times 3 \to 3x - 9y = -12$

① $3x - y = 0$

$-8y = -12 \to y = 1.5$

① $3x - 1.5 = 0 \to x = 0.5$

$\to z = 1/2 + 3/2\, i$

④ sin: $\theta = 360n + \alpha$, $\theta = 360n + (180 - \alpha)$

Key angle (α) $= \sin^{-1}(\cos(20)) = 70$

$70 - 2/3\, x = 360n + 70$, $70 - 2/3\, x = 360n + 110$

$-2/3\, x = 360n$, $-2/3\, x = 360n + 40$

$x = -3/2(360n)$, $x = -3/2(360n + 40)$

$x = -540n$ $x = -540n - 60$

www.mymathscloud.com
My Maths Cloud

(5) a) For denominator → $x = -1$, $x = 2$

As $x \to \infty$, $y \to 0/1$ → $y = 0$

b) $y = -1/2$ → $-1/2 = \frac{x}{(x+1)(x-2)}$

→ $-1 = \frac{2x}{(x+1)(x-2)}$

$-(x+1)(x-2) = 2x$

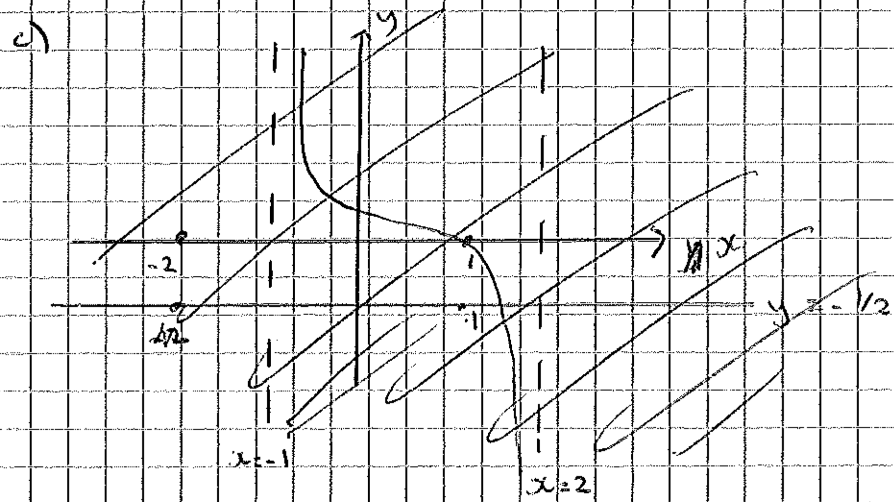
$-x^2 + x + 2 = 2x$

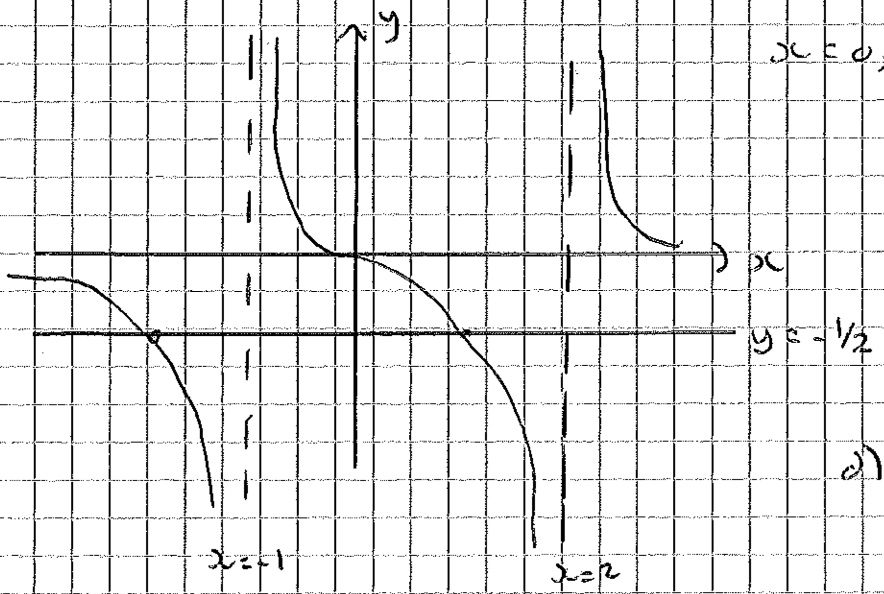
$0 = x^2 + x - 2$

$(x+2)(x-1) = 0$

$x = -2$ $\quad$ $x = 1$

c)





$x = 0, y = 0$

d) curve below $y = -1/2$ when:

$-2 \leq x < -1$

$1 \leq x < 2$

www.mymathscloud.com

6)

a) $\begin{bmatrix} \cos(135) & -\sin(135) \\ \sin(135) & \cos(135) \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

b) i) Looks like a) but with a factor of $\sqrt{2}$

$\rightarrow \sqrt{2} \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$

$\therefore$ Enlargement SF $\sqrt{2}$ & rotation 135^0 anti-clockwise

ii) M^2, scale factor $= (\sqrt{2})^2 = 2$

rotation $= 135 + 135 = 270^0$ anti-clockwise

iii) $\boxed{M^2} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$\boxed{M^4} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$

iv) $M^{2012} = (M^4)^{503}$

$= (-4I)^{503}$

$= [-(2^2) I]^{503}$

$= -2^{1006} I^{503}$ ← $= I$ as I does not change

$= -2^{1006} I \rightarrow n = 1006$

www.mymathscloud.com
My Maths Cloud

(7) a) Let $f(x) = 24x^3 + 36x^2 + 18x - 5$

$f(0.1) = 24(0.1)^3 + 36(0.1)^2 + 18(0.1) - 5 =$

$f(0.2) = 24(0.2)^3 + 36(0.2)^2 + 18(0.2) - 5 = 0.232$

Sign change, ∴ root between 0.1 & 0.2

b)

	Width	Mid-Point	Answer	
0.1 to 0.2	0.1	0.15	-1.409	(-ve)
0.15 to 0.2	0.05	0.175	-0.618875	(-ve)

∴ root lies between 0.175 & 0.2

c) Newton: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 0.2$

$f(x) = 24x^3 + 36x^2 + 18x - 5$

$f(0.2) = 0.232$

$f'(x) = 72x^2 + 72x + 18$

$f'(0.2) = 72(0.2)^2 + 72(0.2) + 18$
$= 32.25$

$\Rightarrow x_{n+1} = 0.2 - \frac{0.232}{32.25}$

$= 0.1934...$ (4dp)

(8) a) $\frac{x^2}{5} + \frac{y^2}{4} = 1$

$\Rightarrow (\sqrt{5}, 0) \quad (-\sqrt{5}, 0) \quad (0, 2) \quad (0, -2)$

b) $\frac{(x-p)^2}{5} + \frac{y^2}{4} = 1$

c) $y = x + 4 \quad \Rightarrow \quad \frac{(x-p)^2}{5} + \frac{(x+4)^2}{4} = 1$

{×20} $\quad 4(x-p)^2 + 5(x+4)^2 = 20$

$4\left[x^2 - 2xp + p^2\right] + 5(x^2 + 8x + 16) = 20$

www.mymathscloud.com
My Maths Cloud

$\Rightarrow 4x^2 - 8xp + 4p^2 + 5x^2 + 40x + 80 = 20$

$\Rightarrow 9x^2 - 8xp + 40x + 4p^2 + 60 = 0$

$\Rightarrow 9x^2 - (8p - 40)x + (4p^2 + 60) = 0$

b) For tangent: $b^2 - 4ac = 0$

$\Rightarrow (8p - 40)^2 - 4 \times (9) \times (4p^2 + 60) = 0$

$\Rightarrow 64p^2 - 640p + 1600 - 144p^2 - 2160 = 0$

$\Rightarrow -80p^2 - 640p - 560 = 0$

$\div -80$ $\Rightarrow p^2 + 8p + 7 = 0$

$(p + 7)(p + 1) = 0$

$\boxed{p = -7}$

$9x^2 \cancel{+9} - (8p - 40)x + 4p^2 + 60 = 0$

$\Rightarrow 9x^2 + 96x + 256 = 0$

$\Rightarrow (3x + 16)(3x + 16) = 0$

$x = -16/3$

$y = x + 4$

$= -4/3$

$(-16/3, -4/3)$

$\boxed{p = -1}$

$\Rightarrow 9x^2 + 48x + 64 = 0$

$(3x + 8)(3x + 8) = 0$

$x = -8/3$

$y = x + 4$

$= 4/3$

$(-8/3, 4/3)$